

# Mathematics as a Second Language

## A Course for Elementary Teachers

*I hate math. And my dad says that's OK because he was never any good at math either!*

— Overheard in a Boston high school

Mathematics is not a beloved subject in America today. Most attitudes range from befuddlement to anxiety to all-out math phobia, and belief in a mythical “math gene” is widespread. Qualified math and science teachers are in short supply, and the pipeline of students into math, science, engineering, and technology is drying up.

The road to math phobia begins in elementary school, where most teachers—reflecting the general population from which they are drawn—have too little knowledge of the subject to teach it effectively. A national curriculum war simmers, but curricular reform is futile unless teachers have deep content knowledge and the confidence to communicate it to students. Most students are turned off by math by the time they reach middle school, where their chances of getting a fully qualified math teacher are slim. By high school, the college-bound group includes precious few students with mastery of the subject or interest in further study. From this pool is drawn the next generation of teachers, continuing our sorry national cycle of math deficiency.

The saddest thing is that all this is unnecessary: math is no mystical subject, inaccessible to all but a chosen few. The key to learning math is *understanding*, which sweeps away both the mystery and the burden of memorization.

The Gross brothers—both mathematicians to the core—collaborated to create *Mathematics as a Second Language* (MSL). This course and textbook are designed to ease teachers and other adults into the world of mathematics and put them on a path to attain the understanding and skills they need to teach and use math effectively.

Herb Gross, recently retired Professor of Mathematics at Bunker Hill Community College, has built a career of making mathematics accessible and understandable to regular people. His brother Ken Gross, Professor of Mathematics at Lesley University and the University of Vermont, founded the Vermont Mathematics Initiative (VMI), where elementary teachers earn master’s degrees by learning and doing real mathematics. MSL was originally designed as the VMI’s first course.

MSL is designed to give participants a deep understanding of mathematics content, set in the context of elementary-school instructional practice. It presents elementary arithmetic—number and operations—as a small, unified, and coherent set of concepts and principles that is at once powerful, beautiful, and fun. It demystifies the language of mathematics, demonstrates the power of logic and reasoning, and focuses on the notorious conceptual blocks involving place value (which few truly understand), fractions/ decimals/percents, rates/ratios/proportions, and negative numbers.

Having taught this remarkably effective course for four years in various Massachusetts school districts, I offer this “executive summary” of how and why it works so well.

—Tom Fortmann

## What is Mathematics?

MSL's paramount goal is to demolish a tragically common misconception—that mathematics is learned by memorizing arcane facts and procedures—and to replace it with deep mathematical *understanding*. The demolition begins with this opening slide, revealed one line at a time:

### **Mathematics**

is a vast collection  
of disconnected facts and procedures  
that don't make much sense  
but must nevertheless be memorized  
(at least long enough to pass the next test).

## **NOT!!!**

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### **Mathematics**

is a small, unified set  
of fundamentally simple concepts and principles  
that anyone can understand and use.

Upon these is built a vast collection  
of results, and rules  
(a few of which should be memorized),  
and applications, and descriptions of our physical world.

And upon these are built  
science, technology, and engineering,  
and indeed the entire infrastructure of modern civilization,  
and most importantly the futures of your students.

## Core Philosophy

The key principles underlying *Mathematics as a Second Language* fall into two categories:

### Mathematical/linguistic principles:

- *Numbers are adjectives that modify nouns.* This seemingly innocuous metaphor is surprisingly powerful. You can add or subtract adjectives if they modify the same noun (inches, dimes, billions, sevenths,  $x$ 's, etc.), and when you multiply or divide adjectives you do the same with the nouns (yielding feet<sup>2</sup>, kilowatt-hours, miles/hour, dollars/pound, etc.). Teachers' evaluations consistently cite this as the single most important idea that they take back and use in their classrooms. It also sneaks in a little rigor (the distributive axiom) without intimidating anyone.
- Numbers have two primary operations: addition and multiplication. The others—*unaddition* and *unmultiplication*—arise from undoing the first two. Interchanging “5–3” with “what you add to 3 to get 5” is a remarkably effective way to enhance understanding, in ways that become apparent when dreaded negative numbers appear on the scene. Similarly, “what do you multiply  $3/5$  by to get  $2/7$ ?” helps demystify that old nemesis, division of fractions.
- To play mathematics, one must accept “*the rules of the game.*” We collect the nine properties of arithmetic one by one on a side board as we encounter them, with frequent references to emphasize that all of arithmetic follows from them. Later, this *rules* theme helps to unify algebra.
- Adjectives and nouns notwithstanding, fractions, decimals, and percents *are just numbers* and have the same properties. We dispel the notion that they are strange hieroglyphics manipulated by arcane rituals, pinpoint their whereabouts alongside the integers on the *number line*, and pose mental-math and estimation problems to develop *number sense*.
- *Arithmetic, geometry, and algebra are all intimately related.* We seize every opportunity to demonstrate this—elucidating the sum  $1+3+5+\dots$ , for instance, by drawing square grids of size 1, 2, 3, ... Other apt examples include linear cost relationships consisting of a fixed fee plus so much per item and, of course, the Pythagorean Theorem.
- *A deep understanding of place value is crucial.* This brilliant invention is the keystone of modern mathematics, science, and technology, but few people truly understand it. We pose probing questions and use exercises in base-5 arithmetic and scientific notation to establish the power and elegance of this topic.

## Pedagogical principles:

- We tell teachers that “this course is for you, not for your students.” They already know how to teach, and we *deepen their understanding* of mathematics so that they can teach it well.
- We provide a *professional environment* and accord teachers the *respect* they deserve.
- We intersperse lectures and presentations with *extensive group and individual work* to provide practice, consolidate learning, build confidence, and explore how to elucidate concepts effectively in a classroom.
- We assign rich, substantive activities and problems without straightforward solutions, in order to stretch the mind, challenge the intellect, and develop *habits of mathematical thinking*.
- We *never show someone how to solve a problem*—we encourage reasoning, suggest approaches, ask for explanations, and offer hints. But mathematical growth and learning come from struggling with problems; the old adage about “teaching a man to fish” applies.
- We emphasize *conceptual understanding*: you have to know what you're doing, why you're doing it, and how it relates to what you've already done. Memorized rules and procedures can be useful, but only if you understand them fully, know how to apply them correctly, and can substitute reasoning when memory fails.
- We eradicate the notion that mathematics consists of disconnected facts and procedures that don't make sense. We show—and demonstrate by our own example—that elementary mathematics is a *unified set of fundamentally simple concepts and principles* that anyone can understand and use.
- To this end, we constantly *use and reinforce* the adjective-noun theme, the number line, the relationship of the operations, and the nine properties of arithmetic.
- Mathematics learning is a staircase. The *next level*—attained only through understanding and mastery of previous levels—provides much greater perspective on the terrain below.

The importance of the last point is manifest in the all-too-common question “Why does an elementary teacher need math beyond the level she teaches?” Prof. Richard Bisk offers an astute riposte: “Suppose that your son's 3<sup>rd</sup>-grade teacher *reads* at the 4<sup>th</sup>-grade level—is that acceptable?”

It's clearly *not* acceptable: effective teachers must understand the mathematics of their own grade levels to a much greater depth than their students, understand subsequent levels that their students will soon encounter, and be able to engage those students in real mathematical discourse.

## A Transformational Experience

Although MSL focuses relentlessly on building *competence* in mathematics, its most important effect is building *confidence*: transforming attitudes, eradicating fears, and leading teachers to the self-surprising realization that “I really can do math!” Back in the classroom, competence and confidence are the keys to engaging students and raising mathematics achievement. This process is eloquently described at the VMI web site ([www.emba.uvm.edu/~gross/vmi](http://www.emba.uvm.edu/~gross/vmi)):

*In the VMI approach to mathematics professional development, participants begin to view themselves as mathematicians, to view mathematics as part of their lives, and to see the world around them in a mathematical light. These transformations take place through a curriculum that is rich in mathematics content, and their impact in VMI teachers' classrooms and schools is far-reaching. The more comfortable teachers feel with mathematics, the more effectively they communicate their knowledge and convey their enthusiasm to their own students and to other teachers.*

This transformation is illustrated by the following quotations from teachers in a recent session of *Mathematics as a Second Language* and its companion course *Functions and Algebra*:

“This course provided me with the understanding for mathematical concepts that I had always known how to compute using ‘faith-based’ mathematics. ... my entire outlook about math has changed. I am confident that my math skills have improved to a level that I can competently explain/teach math classes ...” —*Grade 6-8 Special Ed*

“I have never spent better time in professional development. The last three months have changed my entire attitude toward math and teaching math. Prior to this experience ... when someone mentioned algebra, geometry or graph paper, ... I got a sick feeling and chills, reminding me of being totally lost in my high school and college math classes.”  
—*Grade 1-4 Title I Math*

“I can’t say that this course has made me love math but I no longer fear it!”

“This course has had a profound impact on my teaching. ... I learned so much about the big picture. I gained tremendous confidence and understanding about big ideas in math. I especially appreciated the connections that were reinforced across the strands—I love that and try to build it into my own teaching.” —*Grade 3*

“I entered this course really wanting to increase my own understanding of math so that I would not feel as though I was ‘short-changing’ my students. I also did not want to keep feeling that ‘panicked’ pit-in-my-stomach feeling before math every day. Now, I am excited to teach math *and* I feel a lot more clear as to what to emphasize about where they will go from 4th grade to infinity.”

“... I gained confidence coming to these classes. I know more than I thought and I learned how to express my knowledge and be comfortable with it.”

“I learned *a lot!* ... I no longer feel math-phobic. I can at least start to tackle any math problem. This course ... is a wonderful resource for teachers. I am so happy I could participate.” —*Grade 3*

## Further Information

For further information, see the VMI web site:

<http://www.uvm.edu/~vmi>

or send a note to:

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