The Rationale For This Web Site
(As Seen Through the Eyes of Herb Gross)

An Overview:

It is not uncommon for a person who is not musically gifted to take a course called “Music Appreciation”. Nor does a person have to be an artist to enjoy an Art Appreciation course. It is in this spirit that this web site is dedicated to being a “Mathematics Appreciation” course. You do not have to be either a mathematician or a person whose work involves the use of mathematics in order to appreciate all that mathematics has to offer.

However there is another very important reason that caused me to decide to create this web site. As far back as when I began my teaching career (in the 1950's), mathematics was a stumbling block for many students. Nowadays, however, this problem has become even more severe. Namely in the “good old days” the academic range of abilities for incoming college students was much more narrow. Based on a specific set of admission criteria, teachers could reasonably expect a baseline for the skills of their students. It was an era in which a college education was not perceived as the only measure of human worth in the workplace. A broad menu of occupational choices was available to high school graduates. Depending on individual potential and effort, opportunity existed for a satisfying career, a degree of upward mobility and reasonable economic security.

Today, however, open admissions together with economic, cultural and ethnic diversity, to a degree that has never before existed, have made things dramatically different. In short, the range of academic performance is as diverse as the varied backgrounds of the students themselves. In addition to all of this, entry-level jobs that previously required a high school education now require at least an Associate's degree or some other form of post secondary education. International competition is becoming greater and greater, and the world around us is becoming more complex at a faster and faster rate. The bottom line is that not only has the need for good teaching intensified but we must further recognize that academic coaching strategies that worked as recently as a generation ago are no longer relevant to many of

today's college students. In short, access is no longer a synonym for equal opportunity; and this means that the role of the teacher is even more important today than ever before.

Thus, this site is meant to share with any interested visitors, an approach to the teaching of basic mathematics that has worked well for me.

The "Split Personality" of Mathematics:

It may be said that mathematics, at worst, suffers from schizophrenia; and at best, leads two lives.

• More specifically, most of us understand what is meant when we call mathematics "the hand maiden of science and technology"; and even those of us who profess ignorance of the details of mathematics recognize the role that it plays (and always has played) in the "real world" in which we live.

• We may not realize, however, that if there were a successful "humanistic revolution" and it was decreed that the liberal arts curriculum should rid itself of all courses pertaining to science and technology and, instead, to emphasize only those courses that enhanced the humanities, mathematics would still be a leading course in this new curriculum.

-- For example, Amedeo Avogadro (1776-1856) the Italian chemist and physicist advanced a hypothesis that has come to be called Avogadro's Law.

• Avogadro's law states that equal volumes of all gases under identical conditions of pressure and temperature contain the same number of molecules. From this hypothesis other physicists were able to calculate that there were approximately 6 $x10^{23}$ atoms in a mole of any substance.

Note:
A mole of water weights a little more than half an ounce; and the number 6 $x10^{23}$ when written in place value notation is a 6 followed by 23 zeroes (that is, 600,000,000,000,000,000,000,000)!

Certainly, we have no trouble understanding that this concept clearly belongs to the world of science and mathematics. Yet to grasp the size of 6×10^{23} requires the mindset of a poet! Indeed, if you could count at the rate of one billion atoms per second, it would still take over 18 million years to count that many atoms. Can you name any humanist who had a more vivid imagination? Yet much of our modern theory of the molecular structure of gases rests on the "vivid imagination" of Avogadro!

• When Sir Isaac Newton said that any two objects, no matter how small or how large, exert an influence upon one another; we seem to hear the voice of a poet; or at least a humanist. However, when he adds "with a force directly proportional to the product of their two masses and inversely proportional to the square of the distance between their two centers" we clearly hear the voice of the scientist and mathematician. Moreover, because of Newton's "vivid imagination", we are able to launch space satellites, land on the moon and much, much more.

Note:

Most of us think of the so-called "space age" as having begun in the 1950's. However, it began with Newton in 1683 when he realized that what went up didn't have to come down! All that happened in the 1950's was that we made the technological advances that allowed us to develop a fuel that was capable of giving a rocket the necessary escape velocity to leave the earth's atmosphere and a rocket nosecone covering that prevented the rocket from disintegrating when it reentered the earth's atmosphere. This example serves as an excellent illustration of the interrelationship of theory and technology.

The above two examples illustrate how the world of humanities and world of math/science not only coexist but also are inseparably interwoven; and to study one at the exclusion of the other forever weakens our ability to truly understand both the real world in which we live and the ideal world in which we envision we'd like to live. It is in this spirit that our program has been developed.

-- To study one without the other is like having to decide whether you want to invite your mother or your father - but not both!

-- Or it's like saying “I'd rather be poor but healthy than rich and sick” when in reality you should be saying “I'd rather be rich and healthy than poor and sick”!

Mathematics: A Chronicle of Human Endeavor

It is common for colleges to talk about a “Math/Science department”. Some colleges talk about a Math/Computer-Science department” or a “Math/Statistics department”. But few, if any, colleges talk about a “Math/Humanities department”. Yet, at least in the spirit of our overview, it should be just as natural to talk about “mathematics across the curriculum” as it is, for example, to talk about “writing across the curriculum”; and it is in this spirit that we present “Mathematics: A Chronicle of Human Endeavor”.

It is our claim that by viewing mathematics in a “proper light”, mathematics can be used to enhance the understanding of such topics as philosophy, logic, language, history and even “human nature”. With this in mind, it might be helpful to ask the question “What do we mean by a “chronicle of human endeavor"? More specifically, what is it that separates us from the rest of the animal kingdom? Our answer, which will be stated in general and then applied to mathematics in particular, is summarized by the following three assumptions:

✦ Assumption #1:

We seek the simplest solution to any problem that plagues us; and it is only when this solution becomes either too cumbersome or otherwise outmoded that we look for another solution. Even then we do not look for a complicated solution but rather for the simplest solution we can find that solves the problem.

-- This assumption can be seen ‘in action’ in something as basic as tracing the history of the development of our modern system of enumeration from its primitive inception as a form of sign language to its present powerful but abstract form which we simply call place value.

-- And while this is a good mathematical illustration, it should not be difficult to see that this assumption applies to virtually every field of inquiry. In fact, by making successive refinements to the

world we inherit, we hopefully leave a better world for the ones who follow us. Perhaps this is reflected in a quotation attributed to Sir Isaac Newton when he was called the giant of his century. His reply was "If I could indeed see further than those around me, it was only because I was able to stand on the shoulders of the giants of the preceding centuries".

✦ Assumption #2:

We alone, at least to our knowledge, are the only members of the animal kingdom who are capable of logical thought. That is, we alone are capable of planning and predicting the future based solely on our knowledge of the present and the past. In this regard, mathematics, when understood properly, is an excellent vehicle for helping to understand this gift. Yet in a larger sense, we must remember that the logic used in mathematics is no different from the logic that is used in every physical, natural and social science. The task of deducing inescapable conclusions from given or otherwise arrived at information is a challenge that is found in every field of study.

-- Assumption #2 plays a huge role in mathematics in general, but two applications stand out.

• The ancient Greek, Euclid, is often referred to as "the father of geometry". Yet thousands of years before the birth of Euclid the ancient Egyptians were building pyramids. Certainly, people who could build pyramids must have known about geometry. However, it was Euclid who first codified it into the form of a deductive science. In essence, he started with ten basic assumptions (five of which were called postulates and five of which were called axioms) and derived 14 complete books of inescapable conclusions. In this context, it is the logical structure of Euclidean geometry that has made it a mainstay of the traditional curriculum for over two thousand years.

• While we usually think of geometry as the ultimate model of deductive reasoning, algebra also lends itself to such a study. In fact, many algebra problems are solved by using a handful of basic arithmetical rules to paraphrase more complicated algebraic equations into equivalent, but easier to solve, equations.

✦ Assumption #3:

Despite our drive for material gain and our tendency to place pragmatic pursuits above idealistic values, we are still very capable of appreciating art for the sake of art. In this context, there are facets of almost every subject that transcend "practical applications". There are aspects of mathematics that have an esthetic value that is important quite apart from any practical value. There are mathematicians who study mathematics for the same reason that humanities majors study poetry - because it is beautiful!

-- Indeed, for every page of mathematics that is devoted to helping solve "real world" problems, there are dozens of pages devoted to what we might call "pure" mathematics.

-- For example, of what practical value is it to know that the square of every odd number leaves a remainder of 1 when it is divided by 8?

-- Yet students, many of whom dislike mathematics per se, are intrigued by such facts and are often interested in mathematical "riddles" of the type "Pick a whole number; multiply it by 2; add 1; multiply by 5; cross off the last (units) digit; the answer is the number you picked".

-- In fact, they become so intent on finding an example where the "trick" doesn't work that they do far more computational problems (and at the same enjoy doing them!) than if we had given them a battery of drill problems to do.

• However, we do not mean to imply that there is a need to teach "human values" at the expense of the more traditional subject matter. Rather, it is our belief that when "human values" and subject matter content are taught side-by-side, each emerges more powerfully than it would have if it had been taught at the exclusion of the other. In this respect, the late Samuel Eilenberg made a poignant statement when asked how he juxtaposed his life as a "pure" mathematician with his life as a great "applied" mathematician. His eloquent response was words to this effect: "I compare myself to anesthetic tailor, making jackets as the mood strikes me. Sometimes I make them

> with 32 arms, 55 legs, and 25 different colors. Every now and then, however, I make a conventional coat with two sleeves, no legs and one color; and if a fellow human being comes into my shop and likes the jacket, I give it to him as a favor from the goodness of my heart".

This, in brief, is the driving force behind this web site.